Analysis of Radiocarbon Dating Statistics in Reference to the Voynich Manuscript

The purpose of this article is to identify potential errors in radiocarbon dating with the view to evaluating the Voynich manuscript ^{14}C data. Anyone reading this article should have a basic understanding of statistics.

Potential sources of error in radiocarbon dating are:

1. Type of sample, i.e. protein vs. carbohydrate vs. carbonate.
2. Preparation of sample, i.e. different solvents used for removing surface dirt.
3. Errors from delta ^{13}C isotope dilution measurements.
4. Accelerator Mass Spectrometry background ^{14}C contamination.
5. Counting errors.
6. Terrestrial radiocarbon age calibration curve.

The U. of Arizona corrects their measurements for Delta ^{13}C isotope dilution and for ^{14}C background contamination, Donahue et.al .(*1990 Radiocarbon, vol 32 No. 2, p 135-142*.)

Reimer et.al. (*2004, Radiocarbon, vol 46 No. 3, p 1034-1036*) state that counting statistics do not represent all the uncertainties in radiocarbon dating and that the errors resulting from type of sample, preparation and laboratory differences are difficult to quantify. When the consortium correlated the data resulting from various wood samples from different labs for the 2004 the atmospheric decadal tree ring data set, they applied an **error multiplier k** to the estimated standard deviations (SD) of the various data sets.

$k = \sigma_2/\sigma_1$

σ_2 is the standard deviation in the ^{14}C age of replicate samples with +/- 100 samples analyzed.

σ_1 is the expected standard deviation based on the counting statistics, the average standard deviation of the individual samples σ_i (Stuiver et. al. *Radiocarbon, 1998, Vol. 40 No.3, p.1128*)

The tree ring data set for the years between 1400 and 1500 used Belfast Irish oak. The University of Washington (Seattle data set) supplied most of the measurements. An error multiplier, k = 1.3, was assigned to the Seattle data, additional variability in measurements was assigned to the data sets from Belfast and from Waikato, New Zealand (see Table 1). The offset was determined by inter-laboratory comparisons.

Table 1

Laboratory	Offset	σ_1	σ_2	k	No.samples
Belfast Irish Oak 1986	-6 +/- 1	21.5	29.4	1.37	407
Belfast Irish Oak 2002, 2004	4 +/- 2	21.3	27.6	1.21*	124
Waikato	10 +/- 2	22.0	22.9	1.04*	100

*includes previously applied laboratory error multiplier.

Reimer et.al. recommend that an **error multiplier factor** should be included with all radiocarbon dating results.

A discussion on the reporting ^{14}C data is given Stuiver and Polach (*Radiocarbon, Vol. 19, No.3, 1977, p. 355 – 363.*)

There is no doubt that a more reliable estimate of the 'true' age of the Vinland map would be obtained if additional independent measurements were available. As this is improbable, we are dependent on statistical estimates using current data. The appropriate statistics being:

1. The mean of the values, weighted, according to the individual error estimates, $\mathbf{M_w}$ and unweighted $\mathbf{M_u}$.
2. **n** the number of values.
3. σ_i is the estimated SD for an individual sample measurement.
4. σ_1 is the expected standard deviation based on the counting statistics, the average standard deviation of the individual samples σ_i
5. σ_2 is the standard deviation in the ^{14}C age of replicate samples.
6. σ_w an estimate SD for the weighted mean, $\mathbf{M_w}$, using the individual sample estimates in the weighting. $\sigma_w^2 = 1/\text{sum}\sigma_i^{-2}$ for i = 1 to n
7. σ_u an estimated SD for the population of sample measurements where the individual estimates are ignored. It is usually referred to as an estimate of the standard error of the mean and may be used as an approximation for the total error. $\sigma_u^2 = \sigma_2^2/n$
8. **Chi square** a statistic used to estimate the uncertainties in data without making the additional measurements.

There are limitations to applying statistics to experimental data:

1. Failure to detect systematic errors, this affects the accuracy of the results.
2. Nonstatistical fluctuations in the instruments or the measurements.
3. Carelessness.

$\mathbf{M_w \pm \sigma_w}$ and $\mathbf{M_u \pm \sigma_u}$ values were calculated for five different ^{14}C data sets previously published by the U. of Arizona, for the Vinland map http://www.webexhibits.org/vinland/paper-donahue02.html, and the Shroud of Turin and controls http://www.shroud.com/nature.htm.

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The data and results are given in Table 2.

Table 2

	Vinland map	**Shroud of Turin**	**Sample 2**	**Sample 3**	**Sample 4**
^{14}C BP yr σ_i	338 ± 116	591 ± 30	922 ± 48	1838 ± 47	724 ± 42
^{14}C BP yr σ_i	406 ± 30	690 ± 35	986 ± 56	2041 ± 43	778 ± 88
^{14}C BP yr σ_i	537 ± 51	606 ± 41	829 ± 50	1960 ± 55	764 ± 45
^{14}C BP yr σ_i	486 ± 26	701 ± 33	996 ± 38	1983 ± 37	602 ± 38
^{14}C BP yr σ_i	574 ± 69		894 ± 37	2137 ± 46	825 ± 44
Range	236	110	167	299	223
Chi sq	9	9	9	22	17
df	4	3	4	4	4
Significance	5%	4%	5%	1%	2%
$M_u \pm \sigma_2$	468 ± 96	646 ± 57	925 ± 69	1992 ± 110	739 ± 84
$M_u \pm \sigma_u$	468 ± 43	646 ± 29	925 ± 31	1992 ± 49	739 ± 38
$M_w \pm \sigma_1$	468 ± 58	646 ± 35	927 ± 46	1995 ± 46	721 ± 51
$M_w \pm \sigma_w$	468 ± 17	646 ± 17	927 ± 20	1995 ± 20	721 ± 20
U of A	468 ± 27	646 ± 31	925 ± 32	1995 ± 46	722 ± 43
$k = \sigma_2/\sigma_1$	1.6	1.7	1.5	2.4	1.6
$k_{sher} \sim \sigma_u/\sigma_w$	2.5	1.7	1.6	2.5	1.9

The formulae for calculating the various statistics used in this table are obtainable from any statistics book, I consulted Data Reduction and Error Analysis for the Physical Sciences, *Bevington, P.R. and Robinson, D. 1992.*
BP years before present, i.e. 1950.
df degrees of freedom
$sample_1$ the Shroud of Turin.
$sample_2$ 11-12th century linen.
$sample_3$ linen from Cleopatra's mummy, age measured by scintillation counting = 2,010 ± 80 yr BP.
$sample_4$ cope from St Louis d'Anjou dated at ~ AD 1290 – 1310.

For the Shroud of Turin and controls the radiocarbon ages were calculated using the procedure of Stuiver and Polach,(*1977 Radiocarbon, Vol.19, No. 3, p. 355-363*.) The errors include the statistical (counting) error, the scatter of results for standards and blanks, and the uncertainty in the delta^{13}C determination. The Oxford Radiocarbon laboratory rounded errors in their measurements of the Shroud of Turin and the controls that were below 40 to 40 BP years.

A chi square was calculated for the five data sets in order to determine whether the errors from the individual measurements adequately represented the total error. Table 2 shows that in all cases a chi square of 5% or less was obtained, making it unlikely that the counting errors represent the total error in the radiocarbon dating of the above five samples.

Table 2 also shows that σ_w is considerably less than σ_u. and that an **error multiplier factor, k,** should be applied to each data set. This table also shows that σ_u, the estimate of the standard error of the mean, agrees reasonably well with the estimates the U of Arizona reported for the total errors for the Shroud of Turin and the three controls, but not for the Vinland map. It appears that an additional **error multiplier factor** should be applied to the stated error for the Vinland map. The **k** calculated according to the procedure of Reimer et.al. (*2004, Radiocarbon, vol 46 No. 3, p 1034-1036* emphasize the contribution of the largest error measurement to $\boldsymbol{\sigma_1}$, thereby making the corresponding **k** value smaller. The method I use sensibly de-emphasizes the larger errors and in turn makes the $\mathbf{k_{sher}}$ value larger.

Table 3 shows a comparison of the mean ^{14}C BP years reported by U of Arizona for the four Shroud of Turin samples with those reported by the two other laboratories of equal prestige.

Table 3

Sample	**Shroud of Turin**	**Sample 2**	**Sample 3**	**Sample 4**
Arizona	646 ± 31	927 ± 32	1,995 ± 46	722 ± 43
Oxford	750 ± 30	940 ± 30	1,980 ± 35	755 ± 30
Zurich	676 ± 24	941 ± 23	1,940 ± 30	685 ± 34
Chi sq (2df)	6.4	0.1	1.3	2.4
Significance %	5	90	50	30

The U of Arizona and the U of Oxford show a difference of 104 years in their reported mean ^{14}C BP years for the Shroud of Turin. The chi square (2 degrees of freedom) calculated for this data is 6.4 with a level of significance of 5%. The probability that random error alone is responsible for the scatter between the results reported by three labs is less the 5%. This indicates the possibility of the presence of systematic errors in the radiocarbon measurements for the Shroud of Turin, perhaps due to different sample preparation methods.

The need to provide the best possible data for converting radiocarbon ages into calendar ages resulted in an update in the atmospheric decadal tree ring data set in 2004, Reimer et.al. (*Radiocarbon, Vol. 44, No. 3, p. 1029-1056*). The Supplemental Data on which this curve is based may be found at http://www.radiocarbon.org/IntCal04.htm. I have extracted a portion of the data used to construct the terrestrial radiocarbon age calibration curve for the years 1400 – 1500 A.D. from:
http://www.radiocarbon.org/IntCal04%20files/intcal04.14c,

Table 4

Calendar age	BP age	^{14}C BP age	error yr ^{14}C BP age
1380	570	646	12.0
1385	565	628	12.0
1390	560	604	13.0
1395	555	583	12.0
1400	550	572.0	12.0
1405	545	560.0	11.0
1410	540	543.0	12.0
1415	535	526.0	12.0
1420	530	513.0	13.0
1425	525	502.0	12.0
1430	520	491.0	13.0
1435	515	477.0	12.0
1440	510	459.0	13.0
1445	505	438.0	12.0
1450	500	415.0	12.0
1455	495	398.0	11.0
1460	490	393.0	12.0
1465	485	389.0	12.0
1470	480	381.0	13.0
1475	475	374.0	12.0
1480	470	372.0	13.0
1485	465	368.0	12.0
1490	460	359.0	12.0
1495	455	352.0	11.0
1500	450	350.0	12.0

*The error in the ^{14}C BP age represents 1 SD

Radiocarbon dating of four pages of the Voynich Manuscript has provided a 2SD range of 1404 – 1438. Mean 1421

1. Using Table 4, I extrapolated the ^{14}C BP year values of 565 and 473, for the calendar years 1404 and 1438 respectively.
2. Total σ = (565 –473)/4 (1SD)
3. Total σ = $[(\text{sample } \sigma)^2 + (\text{curve } \sigma)^2]^{1/2}$ (*Stuiver and Becker 1993, Radiocarbon, Vol.35, No.1, p. 39*.) [$24^2 = (\text{sample } \sigma)^2 + 12^2$]
4. Sample σ = 21 BP yrs.
5. Mean ^{14}C BP year = 514 +/- 21 (1SD)

If the individual ^{14}C BP year values for the four pages of the Voynich Manuscript are available, it is possible from the variance in their ages to calculate an estimate of the standard error of the mean. This estimate would indicate whether an error multiplier factor should be applied to the 1SD error of 21.

Table 5 shows the raw data used to obtain Table 4.
http://www.radiocarbon.org/IntCal04%20files/IntCal04_rawdata.csv.
The raw data was converted to the atmospheric decadal tree ring data set using a random walk model (*Buck and Blackwell, 2004, Radiocarbon, Vol.46, No.3, p 1093-1102*)

Table 5

Cal age	BP age	14C BP age	14C BP age error	N	Laboratory
1389	561	637.6	18.8	946	Seattle
1389	561	627.9	18.9	947	Seattle
1389	561	651	23	108	Belfast 2002
1389	561	623	18.7	57	Waikato
1399	551	574.7	18.6	948	Seattle
1399	551	574	23	107	Belfast 2002
1399	551	599	46.6	175	Belfast 1986
1399	551	588	18.7	56	Waikato
1409	541	564.5	17.3	949	Seattle
1409	541	577	14.5	106	Belfast 2002
1409	541	556	18.7	55	Waikato
1419	531	520.3	18.1	950	Seattle
1419	531	539	21.8	105	Belfast 2002
1419	531	512	19.2	174	Belfast 1986
1419	531	536	18.7	54	Waikato
1429	521	499.5	17.4	951	Seattle
1429	521	511	23	104	Belfast 2002
1429	521	499	18.7	53	Waikato
1439	511	468.2	18.6	952	Seattle
1439	511	484	20.6	103	Belfast 2002
1439	511	500	26	173	Belfast 1986
1439	511	488	17.7	52	Waikato
1449	501	417.7	15.3	953	Seattle
1449	501	457	20.8	51	Belfast 2002
1459	491	388.5	18.2	954	Seattle
1459	491	389.5	17.8	955	Seattle
1459	491	415	25.4	101	Belfast 2002
1459	491	458	23.3	172	Belfast 1986
1459	491	377	17.7	50	Waikato
1469	481	395	18.7	956	Seattle
1469	481	397	21.8	100	Belfast 2002
1469	481	406	13.7	171	Belfast 1986
1469	481	375	17.7	49	Waikato

The ^{14}C BP year measurements were averaged into 10 yr bins**, the cal ages are given at the starting year of the cal year span** http://www.radiocarbon.org/IntCal04%20files/IntCal04%20raw%20datasets.pdf.

Table 5 shows that the Seattle raw data age estimates tend to be younger (positive offset) than the other data sets (Reimer et.al. *2004, Radiocarbon, Vol.46 No.3, p.10345*). This indicates the presence of a systematic error between data from Seattle and data from Belfast and Waikato. I have not been able to find any information indicating what the offset might be in ^{14}C dating between U of Arizona and the 2004 atmospheric decadal tree ring data set.

It should be remembered that ^{14}C dating measures sample activity not sample age and that the conversion from ^{14}C BP age to calendar age is dependent on the use of the atmospheric decadal tree calibration curve with its own set of limitations with respect to accuracy and precision. I am concerned by the large variation between the decadal ^{14}C data from the three labs for the years from 1419 to 1459 in Table 5, years critical to the dating of the Voynich Manuscript. The three labs made a large number of measurements on samples of Belfast Irish oak of known age and this data was used to produce the 2004 atmospheric decadal tree ring data set.

In conclusion, until a better method becomes available, radiocarbon dating is the best method for determining the approximate age of small samples of organic material. The Oxford Radiocarbon laboratory seems to believe that a realistic estimate of the S.D. of the ^{14}C BP age should not be less than 40 years http://www.shroud.com/nature.htm and additionally they do not accept responsibility for any financial loss as a result of an erroneous report: http://www.shroud.com/vanhelst.htm. Having reviewed the available data and taking into account the variety of possible errors in ^{14}C dating, I have come to my personal conclusion that the animal(s) whose skins were used to make the parchment for the Voynich Manuscript were probably killed some time during the first half of the 15th century.

Error is a normal part of science, no method is immune, results should be subjected to a critical examination and control experiments performed to determine the accuracy of the measurements. Finally it never hurts to review the literature, this should always be the initial step in any endeavor.

The references quoted in this paper can be found on the Radiocarbon web site http://www.radiocarbon.org/.